ASSIGNMENT #8

Please do not write your answers on a copy of this assignment, use blank paper. As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. In addition to the true and false section being graded, I will grade one other problem; this will account for 10 points out of 25. The other 15 will be based on completion. **If you would like feedback on a particular problem, please indicate it somehow.** You must make an honest attempt on each problem for full points on the completion aspect of your grade.

(1) Find bases for the following subspaces. Some of these spaces may look familar...

  (a) The subspace $\{(v_1, v_2, 0) \mid v_1, v_2 \in R\}$ of $\mathbb{R}^3$.

  (b) The vector space $\mathbb{R}[x]_{\leqslant 4}$.

  (c) Let A be any $m \times n$ matrix. Is the image of the linear transformation $T_A : \mathbb{R}^n \to \mathbb{R}^m$ (recall T_A is multiplication by A on the left) a subspace of $\mathbb{R}^m$?

  (d) Is the set of points inside and on the unit circle in $\mathbb{R}^2$ a subspace of $\mathbb{R}^2$? *Hint:* the set of points inside on the unit circle can be described as $H = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leqslant 1\}$?

  (e) The set of polynomials of $\mathbb{R}[x]_{\leqslant 21}$ that has 1 and 2 as roots.

  (f) The subspace $V = \{ax^2 \in \mathbb{R}[x] \mid a \in \mathbb{R}\}$ of $\mathbb{R}[x]_{\leqslant 2}$.

(2) Using your answers to problem (1), find the dimensions of the following spaces.

  (a) The subspace $\{(v_1, v_2, 0) \mid v_1, v_2 \in R\}$ of $\mathbb{R}^3$.

  (b) The vector space $\mathbb{R}[x]_{\leqslant 4}$.

  (c) Let A be any $m \times n$ matrix. Is the image of the linear transformation $T_A : \mathbb{R}^n \to \mathbb{R}^m$ (recall T_A is multiplication by A on the left) a subspace of $\mathbb{R}^m$?

  (d) Is the set of points inside and on the unit circle in $\mathbb{R}^2$ a subspace of $\mathbb{R}^2$? *Hint:* the set of points inside on the unit circle can be described as $H = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leqslant 1\}$?

  (e) The set of polynomials of $\mathbb{R}[x]_{\leqslant 21}$ that has 1 and 2 as roots.

  (f) The subspace $V = \{ax^2 \in \mathbb{R}[x] \mid a \in \mathbb{R}\}$ of $\mathbb{R}[x]_{\leqslant 2}$.

(3) Write down two different bases for $\mathbb{R}^3$ that are not the standard basis (remember, the standard basis for $\mathbb{R}^3$ is $\{(1, 0, 0), (0, 1, 0), (0, 0, 1\})$.

(4) Find a basis for the subspace of $\mathbb{R}_{\leqslant 2}[x]$ spanned by the vectors $-1 + x - 2x^2, 3 + 3x + 6x^2, 9$.

(5) For each of the linear transformations $T : V \to W$ of vector spaces below, i) find a basis for $\ker(T)$, ii) find nullity(T), and iii) use the rank-nullity theorem to find rank(T).

(a) $T : \mathbb{R}^3 \to \mathbb{R}^4$, where $T(\mathbf{x}) = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 7 \end{bmatrix}$.

(b) $T : \mathbb{R}[x]_{\leqslant 3} \to \mathbb{R}[x]_{\leqslant 2}$, where $T(f) = \frac{df}{dx}$.

(6) For each of the following matrices, find a bases for it's null space, column space, and row space.

(a) $\begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

(7) Answer the following true and false questions. You do not need to provide justification.

(a) The vector space $\mathbb{R}[x]_{\leqslant n}$ is isomorphic to $\mathbb{R}^{n+1}$.

(b) The vector space $\mathbb{R}^3$ is isomorphic to $\mathbb{R}^2$.

(c) Let V be a vector space with two subspaces (not necessarily the same) U and W. The subset $U \cup W := \{v \in V \mid v \in U \text{ or } v \in W\}$ is a subspace of V.

(d) There is a set of 11 vectors that span $\mathbb{R}^{17}$.

(e) The kernel of a linear transformation $T : V \to W$ is a subspace of V.

(f) The image of a linear transformation $T : V \to W$ is not a subspace of W.